

B. Math. III Topology Mid-semester Examination 2013

Write “True” or “False” for each question on the *FIRST PAGE* of your answer book. *DO NOT* write anything else on the first page, which is the *ONLY* page that will be graded. Each correct answer merits 5 marks, and each wrong answer gets 0 marks. Attempt all questions. You may do your rough work on remaining pages or extra sheets (none of which will be graded). You may use books and notes.

1. There exists a sequence $\{x_n\}$ in $\overline{S_\Omega}$ with $x_n < \Omega$ for all n , which converges to Ω . (True/False)
2. The map $f : (\mathbb{R}, \mathcal{T}) \rightarrow (\mathbb{R}, \mathcal{T})$ defined by $f(x) = -x$ is continuous for every topology \mathcal{T} on \mathbb{R} . (True/False)
3. The subset $F := \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\} = K \cup \{0\}$ is a compact subset of \mathbb{R}_K . (True/False)
4. The map $f : S_\Omega \rightarrow S_\Omega$ defined by $f(x) = x_+$ is continuous. (True/False)
5. If $f : X \rightarrow Y$ is a continuous map of topological spaces, then $f^{-1}(\text{int } A) \subset \text{int } f^{-1}(A)$ for every $A \subset Y$. (True/False)
6. Let \mathbb{R} have the usual topology, and let X denote $[0, 1] \times [0, 1]$ with the dictionary order topology. Then there exists a continuous surjection $f : \mathbb{R} \rightarrow X$. (True/False)
7. Let \mathbb{R}_l denote the reals with lower limit topology, and let $\mathbb{R}_l \times \mathbb{R}_l$ have the product topology. Then the map $f : \mathbb{R}_l \times \mathbb{R}_l \rightarrow \mathbb{R}_l$ defined by $f(x, y) = xy$ is continuous. (True/False)
8. Let X be the topological space defined by giving the cofinite topology to the set \mathbb{R} . Then X is path-connected. (True/False)